

## 09 Systems of equations

### ENCRYPTION

Based on : 'Classical Encryption Techniques' By Omar Gameel Salem  
<http://www.codeproject.com/Articles/63432/Classical-Encryption-Techniques>

Encryption is a method which allows information (for example, a secret message) to be hidden so that it cannot be read without special knowledge (such as a password).

Once this is done the information is encrypted.

Decryption is a way to change an encrypted piece of information back into unencrypted form. This is called the decrypted form.

One of the basic building blocks of all encryption techniques is substitution: letters of text are replaced by other letters or by numbers or symbols.

#### ***Caesar Cipher***

The earliest known substitution cipher was used by Julius Caesar.

It involves replacing each letter in the plaintext by a shifted letter in the alphabet used.

Example: If the shift value is (3) then we can define transformation as:

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

so the message ***"meet me after the toga party"*** becomes:

PHHW PH DIWHU WKH WRJD SDUWB

We can have only 26 trials!!

Could you break cipher text? "GCUA VQ DTGCM" (it is in English) –the shift value is 2-.

#### ***Mono-alphabetic Cipher***

Each plaintext letter maps to a different random cipher text letter, hence the key size is 26 letters long. Example:

Key:

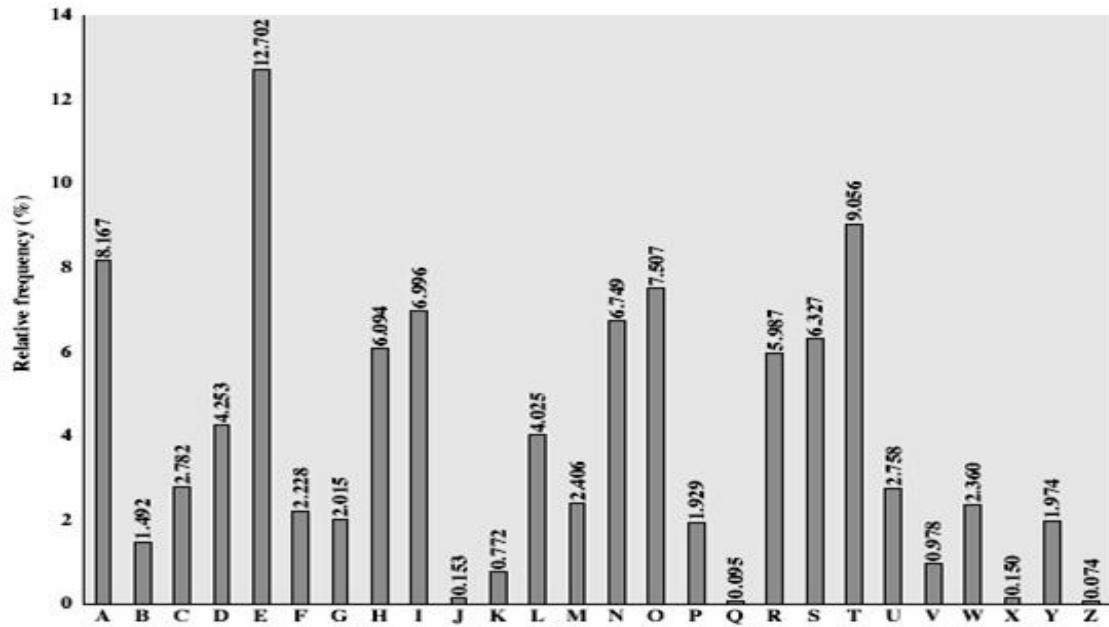
a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
D	K	V	Q	F	I	B	J	W	P	E	S	C	X	H	T	M	Y	A	U	O	L	R	G	Z	N

Plain text: ifwewishtoreplaceletters

Cipher text: WIRFRWAJUHIFTSDVFSFUUFYA

Now the Brute Force attack to this cipher requires exhaustive search of a total of  $26! = 403291461126605635584000000$  keys, but they can use the frequency of the characters to discover the equivalence and then the shift rule.

This is a graphic that shows the frequency in English language.



## THE KEYBOARD

The frequency of the letter in English was the reason to distribute the positions in a keyboard. But, curiously, there are some not very used letters in the best positions. For example, j; k; ñ, f.

If you would have to design a keyboard, what 8 letter will you place in the principal finger position?

What is the reason why the keyboard has other letters that are not very frequent? Do you know?



## 1. DEFINITIONS

An equation is linear if it is a first-degree equation. For example,  $x + y = 10$ .

The solutions are an infinite set of pairs, one for each unknown, that make all the equalities right.

For example, two numbers add up to 12.

Write the equation that expresses this situation.

Write several solutions.

A system of equations is a set of equations referred to the same unknowns.

The solution of a system of equations is the set of values that get every equality right.

For example, two numbers add up to 12 and their difference is 6.

Write the system that defines this problem.

Write the solution for it.

Two systems are equivalent if they have the same solutions.

Resolution is the method we use to find the solutions. The aim for every method is to get an equation with one unknown.

## 2. SUBSTITUTION OR REPLACEMENT METHOD

It consists of clearing an unknown in one of the equations and replacing it in the other one.

This method has four steps.

Follow the steps on the right to solve this system: 
$$\begin{cases} x + y = 5 \\ 2x - y = 7 \end{cases}$$

First step: We clearing one unknown in one of the equations.	
Second step: We substitute the previous value in the corresponding unknown of the other equation.	
Third step: We solve the resulting equation.	
Fourth step: We work out the other unknown by the first step expression.	

### 3. EQUALIZATION METHOD

The equalization method consists of clearing the same unknown in both equations and then equalizing the obtained expressions.

So we have to follow four steps.

Follow the steps on the right to solve this system: 
$$\begin{cases} 2x + y = 2 \\ -x + y = 1 \end{cases}$$

First step: We clearing the same unknown in both equations.	
Second step: We equalize both expressions.	
Third step: We solve the resulting equation.	
Fourth step: We calculate the other unknown by one of the first step expressions.	

#### 4. REDUCTION OR ELIMINATION METHOD

This method uses the properties of equalities in order to get two equations with two opposite coefficients for the same unknown.

Then we add one equation to the other and we get an equation with only one unknown. Our goal is achieved!

We have to do the following steps.

Follow the steps on the right to solve this system: 
$$\begin{cases} 2x - 3y = 18 \\ -5x + 2y = -23 \end{cases}$$

First: We multiply every equation by a suitable number in order to get two opposite coefficients.	
Second: We add one equation to another.	
Third: We solve the resulting equation.	
Fourth: We work out the other unknown by one of the initial equations.	

## 5. PROBLEM RESOLUTION

The way to solve a problem is to translate the problem into algebraic language. For example, we are going to solve the following problem:

I paid €24 for a 2 kg package of nuts. The package is a mixture of almonds and raisins. If the almonds are €16 per kilo and the raisins are €6 per kilo. How many kilos does the package have of each?

Follow the steps on the right to solve this problem:

First: We locate the unknown values and we name everyone with a different letter or unknown.	
Second: We get the equations by translating the instructions into symbolic language.	
Third: We solve the resulting system of equations.	
Fourth: We answer the problem questions according to the result.	

## PROBLEMS AND EXERCISES

### 1. DEFINITIONS

1. Write four solutions for the equation  $x + y = 10$ .
2. Write four solutions for the next equation:  $x + 2y = 5$
3. Check that  $x = 2, y = 4$  is a solution for  $\begin{cases} 2x + y = 8 \\ x - y = 1 \end{cases}$  without solving the system.
4. Invent a system of equations whose solution is  $x = 4$  and  $y = 3$

### 2. SUBSTITUTION OR REPLACEMENT METHOD

5. Solve the system by the substitution method  $\begin{cases} x = 2y + 5 \\ 3x - 2y = 19 \end{cases}$
6. Solve the system by the replacement method:  $\begin{cases} 2x + 3y = -1 \\ 3x + 4y = 0 \end{cases}$
7. Solve the following system by the substitution method:  $\begin{cases} 3x + 4y = 16 \\ 5x - 9y = 11 \end{cases}$

### 3. EQUALIZATION METHOD

8. Solve by the equalization method and check the result:  $\begin{cases} 2x - 2y = -16 \\ -3x + 2y = 16 \end{cases}$
9. Solve the following system by the equalization method:  $\begin{cases} 2x + 3y = 12 \\ 5x - 7y = 1 \end{cases}$

### 4. REDUCTION OR ELIMINATION METHOD

10. Solve the system by reduction method:  $\begin{cases} 4x + y = 5 \\ 3x + 4y = -6 \end{cases}$
11. Solve the system by reduction method:  $\begin{cases} 5x + 3y = 12 \\ 7x - 6y = 27 \end{cases}$
12. Solve the system by a suitable method  $\begin{cases} \frac{x}{3} + \frac{y}{2} = \frac{1}{2} \\ -x - 2y = 2 \end{cases}$  (first remove the denominators)
13. Solve the system by a suitable method  $\begin{cases} \frac{3}{2}x - 2y = 2 \\ 3x - \frac{5}{2}y = 7 \end{cases}$  (first remove the denominators)
14. Solve by the reduction method  $\begin{cases} \frac{2}{5}x - y + 3 = 0 \\ -x + 2y - 5 = 0 \end{cases}$  (first take away the denominators)



- 15.** Solve the system by the three methods: substitution, equalization and reduction  
method:  $\begin{cases} x - 2y = 5 \\ 3x - 2y = 19 \end{cases}$

**5. PROBLEM RESOLUTION**

- 16.** What is the area of a rectangle knowing that its perimeter is 16 cm and its base is three times its height?
- 17.** What are the sides of a rectangle knowing that its base is three times its height and the surface area is 48 cm<sup>2</sup>?
- 18.** A hotel has 50 rooms and 87 beds. Knowing that there are only single rooms and double rooms, how many rooms are there of each type?
- 19.** A hotel has 45 rooms and 115 beds. Knowing that there are only double rooms and triple rooms, how many rooms are there of each type? Write the equations and then solve the system.
- 20.** John purchased a computer and a TV for \$2,000 and later sold both items for \$2,260. How much did each item cost, knowing that John sold the computer for 10% more than the purchase price, and the TV for 15% more?
- 21.** A farm has pigs and turkeys, in total there are 58 heads and 168 paws. How many pigs and turkeys are there?
- 22.** John says to Peter, "I have double the amount of money that you have" and Peter replies, "if you give me six dollars we will have the same amount of money". How much money does each have?
- 23.** The price of two appliances is \$3,500. But the first item has a 10% discount and the second a 8% discount, then the final price is \$3,170. What is the original price of each item?
- 24.** Find a two-digit number knowing that its digit in the tens column minus 5 is the same digit in the ones column and if the order of the digits is reversed, the number obtained is equal to the first number, minus 27.
- 25.** 3 kilos of oranges and 2 kilos of apples cost €9 while 2 kilos of oranges and 2 kilos of apples cost €7, what is the cost of each fruit per kilo? (Write the equations that represent the situation and solve the system.)
- 26.** A palindromic number is one which you read the same from right to left as left to right. For instance, 54745.  
A palindromic number has three digits, that is  $xyx$ , and we know that the tens digit ( $y$ ) is double the hundreds digit ( $x$ ) and the three digits add up to 16. What is the number?